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Math of the Ellipse

Technical Article:

DETERMINATION OF IMPACT ANGLE USING MATHEMATICAL PROPERTIES OF THE ELLIPSE

INTRODUCTION

The mathematical relationship between the impact angle of a blood droplet on a surface and the width and length ratio of the resultant bloodstain has been a long accepted principle of bloodstain pattern analysis. This relationship is often stated as,

$$SINE\ \theta = \frac{Width}{Length} \quad \text{(or abbreviated as } SIN\ \theta = \frac{Width}{Length}\text{).}$$

One of the critical skills for the student of bloodstain pattern analysis is to make proper measurements of the bloodstains width and length in order to calculate the proper impact angle. Improper measurement can greatly alter what is an estimate with respect to the impact angle. The traditional method, taught at many basic courses, involves envisioning a perfectly fitted ellipse superimposed on the bloodstain. No excess portions such as tails or spines, outside the boundaries of the ellipse should be included in the measurement determining the length and width. (See Figure –1.)

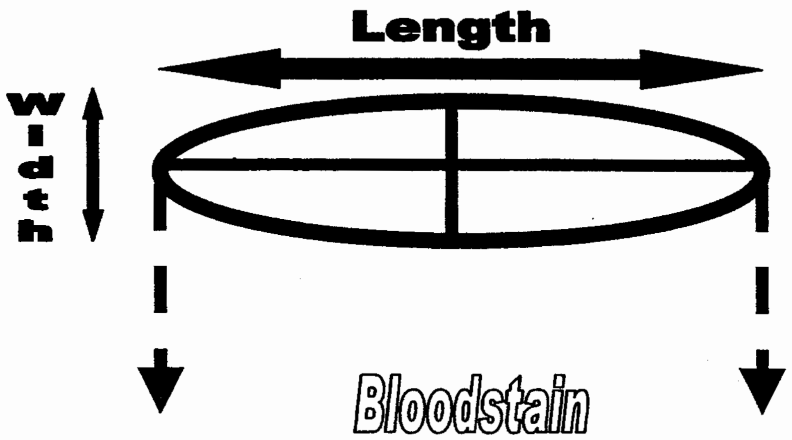

Figure –1

To determine the appropriate length and width measurements the analyst must envision a perfectly fitted ellipse superimposed on the bloodstain.

By selecting well-defined stains it is often easy to determine the width of a bloodstain. The excess tail portion may present some difficulty for beginning students in determining the length as it maybe difficult to envision where the vertex of the perfectly fitted ellipse falls within the extended tail region.

By exploring some mathematical properties associated to the general ellipse, we discovered additional methods of determining the ratio associated to the sine of the impact angle of a bloodstain.

GEOMETRY OF AN ELLIPSE

An ellipse is the set of all points, in a plane, the sum of whose distance from two fixed points (the foci) is constant. The standard general equation for an ellipse, with the length parallel to the x-axis and centered at the origin, can be stated as:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

where the numeric values associated to "a" and "b" of this equation have a relation to the length and width of the ellipse. (See Figure –2.)

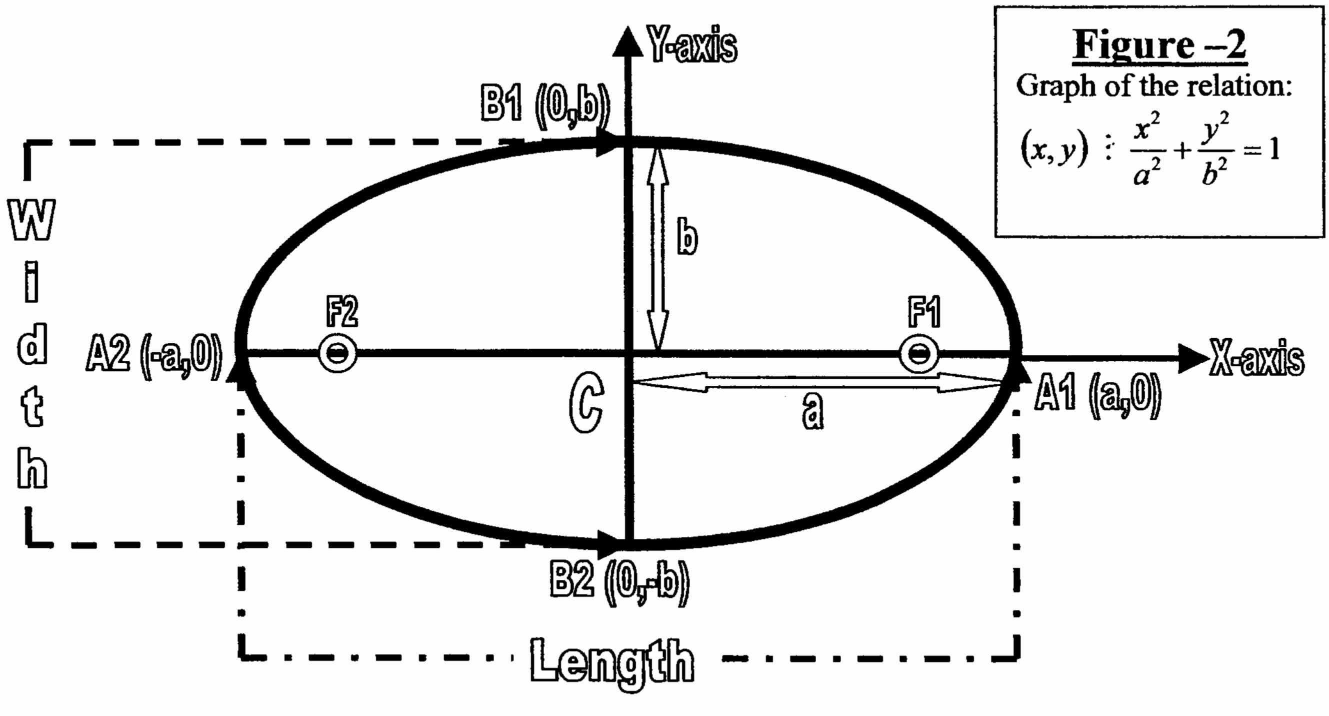

Figure –2
Graph of the relation:
$(x,y) \vdots \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

In Figure – 2 the points A1, A2, B1, and B2 are called the vertices of the ellipse. The points F1 and F2 are the foci (plural of "focus") of the ellipse.

The line segment $\overline{A1A2}$ is called the major axis or more commonly the length by the bloodstain analyst. The midpoint of $\overline{A1A2}$ is "*C*" the center of the ellipse.

The line segment $\overline{B1B2}$ which is perpendicular to the major axis and through the center ("*C*") of the ellipse is called the minor axis, or the width by the bloodstain analyst.

The distance from the vertex A1 to the center, is called the semi-major axis ("a"). Similarly the distance from the vertex B1 to the center of the ellipse, is called the semi-minor axis ("b").

APPLICATION TO BLOODSTAIN PATTERN ANALYSIS

Note that the major axis (or length) is equal to "2a" while the measurement of the minor axis (or width) is equal to "2b." We can substitute this information into the trigonometric equation involving the width and length to obtain:

$$SIN\ \theta = \frac{Width}{Length} = \frac{2b}{2a} = \frac{b}{a}$$

Thus the impact angle of a blood droplet can also be defined equally by the width / length ratio or by the semi-minor / semi-major ratio. This offers the analyst additional methods of determining the impact angle. One such method is by simply measuring the semi-minor and semi-major axis, and calculating a direct ratio. For example if a bloodstain had a semi-minor axis of 3 mm and a semi-major axis of 4 mm, this could be used to determine:

$$SIN\ \theta = \frac{3}{4} = 0.75 \Rightarrow \theta = SIN^{-1}\left(0.75\right) = 48.59^{\circ}$$

We could also determine the ratio associated with an impact angle by taking the actual width of the bloodstain and by doubling the measurement of the semi-major axis, that is:

$$SIN\ \theta = \frac{Width}{2a}$$

If we use the same data as above, the width would be 6 mm and the semi-major axis would be 4 mm and we obtain:

$$SIN\ \theta = \left(\frac{6}{2(4)}\right) = \left(\frac{6}{8}\right) = \left(0.75\right) \Rightarrow \theta = SIN^{-1}\left(0.75\right) = 48.59^{\circ}$$

Figure –3 demonstrates this concept on a bloodstain that impacted the target surface at a 30° angle. The first method demonstrates the semi-minor to semi-major axis ratio while the second method expresses the width over twice the semi-major axis ratio. Either method employed obtains the same angle of impact calculation.

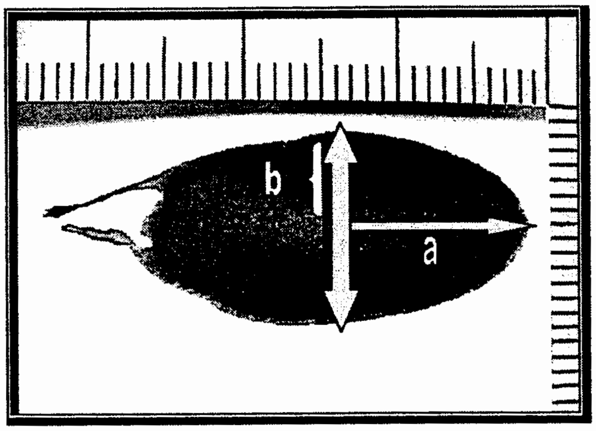

Figure – 3

Bloodstain where b = 6mm and a = 12 mm.

$$(1)\quad SIN\ \theta = \frac{b}{a} = \frac{6}{12} = 0.5$$

or

$$(2)\quad SIN\ \theta = \frac{Width}{2a} = \frac{12}{2(12)} = \frac{12}{24} = 0.5$$

$$\theta = \left(SIN^{-1}\right)\ 0.5 = 30^{\circ}$$

CONCLUSION

This method maybe useful when one has difficulty determining the vertex of an elliptical bloodstain due to obscurity by the tail or spine. By using the half of the ellipse that is better defined, more accurate measurements may result. The beginning student might find this method easier and more accurate than the traditional technique. The experienced analyst who does not have access to current software programs (such as Dr. A. L. Carter's "Back Track ®" – Forensic Computing of Ottawa) for reconstruction purposes may utilize these techniques as an alternative or supplement to the traditional methodology.

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