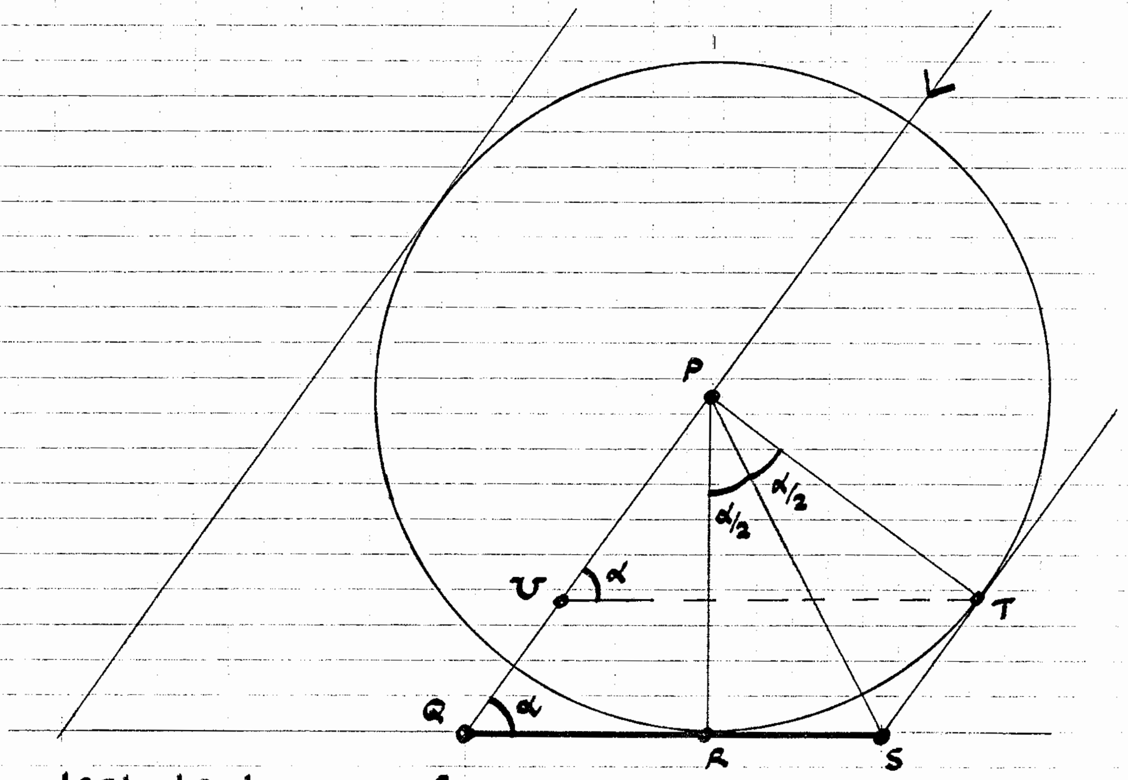
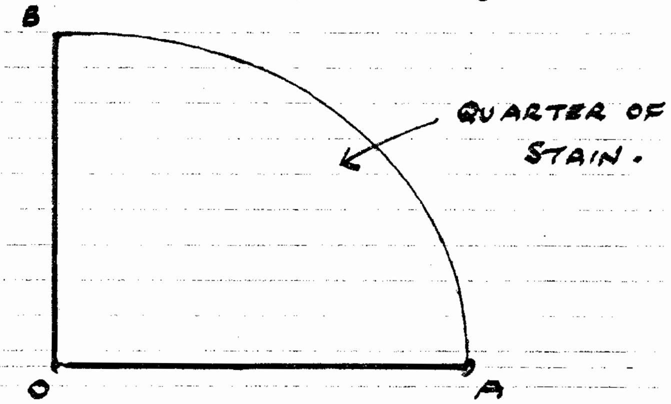
JUSTIFICATION OF THE SINE FORMULA

$$\sin \alpha = \frac{b}{a}$$



$|RS| = |ST|$



ASSUME $|OB| = r$

$|OA| = a, \quad |OB| = b$

In ΔPQR $\quad |PR| = r$ = radius of sphere.

$$\sin\alpha = \frac{|PR|}{|PQ|} = \frac{r}{|PQ|} = \frac{b}{|PQ|}.$$

want to show $|PQ| = |QS| = a$

draw UT parallel to QS.

Because $QSTU$ is a parallelogram

$|QS| = |UT|$. Also $\angle TUP = \alpha$

In ΔPQR, $\sin\alpha = \dfrac{|PR|}{|PQ|} = \dfrac{r}{|PQ|}$

$$\Rightarrow |PQ| = \frac{r}{\sin\alpha} = r\operatorname{cosec}\alpha$$

And in ΔPUT, $\sin\alpha = \dfrac{|PT|}{|UT|} = \dfrac{r}{|UT|}$

$$\Rightarrow |UT| = \frac{r}{\sin\alpha} = r\operatorname{cosec}\alpha$$

$\therefore |PQ| = |UT|$ $\quad \therefore a = |QS| = |UT| = |PQ|$

Shows $|PQ| = a$, and so $\boxed{\sin\alpha = \dfrac{b}{a}}$